International Baccalaureate

Internal Assesment

Physics

Investigating Luminance in LEDs

How does a change in voltage affect luminance of a LED?

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Nomenclature

Universal Constants

e	Euler's Number	
k	Boltzmann's Constant	$1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
q	Fundamental Charge	$1.602\,176\,634 \times 10^{-19}\,\mathrm{C}$

Assumed Constants

η	Luminous efficacy	$90.9\,\mathrm{lm\,W^{-1}}$
A_{sensor}	Area of the camera sensor	$7.5 \times 10^{-4}\,\mathrm{m^2}$
I_S	Saturation Current	$5.591\,105\,779\,517\,251 \times 10^{-4}\,\mathrm{A}$
n	Ideality Factor/Emission Coefficient	90.90
T	Temperature	$300.05\,\mathrm{K}$
V_T	Thermal Voltage	$2.585\,630\,845 \times 10^{-2}\,\mathrm{V}$

Other Symbols

λ	Wavelength	m
ω	Beam waist	m
ϕ_{lm}	Useful luminous flux	lm
τ_P, τ_N	Carrier lifetimes of holes and electrons, respectively	s
θ	Divergence angle	rad
A	Cross-sectional area	$\mathrm{m^2}$
D_P, D_N	Diffusion coefficients of holes and electrons, respectively	$\mathrm{m^2}$
E_V	Illuminance	lx
I, I_D, I_{rms}	Current across a diode	A
I_0	Peak Intensity	$\mathrm{W\,m^{-2}}$
N_D, N_A	Donor and Acceptor concentrations at the n and p side, respectively	$\mathrm{atoms\,m^{-3}}$
n_i	Intrinsic carrier concentration in the semiconductor material	$\mathrm{carriers\,m^{-3}}$
P, P_{rms}	Power consumed by the diode	W
V, V_D, V_{rms}	Voltage across a diode	V

1 Introduction

When I was a child, My parents had adapted a mount of a ceiling fan for a light fixture. As the rest of the wiring was unmodified, the regulator, which was used to control the speed of the fan. Almost intuitively, I turned the regulator (which I later learnt was a potentiometer) to see if it would dim the light, like it used to slow down the fan. To my surprise, the light actually dimmed. Now that I have grown up and taken then the IB, with an aspiration to delve into Engineering, I sought to model the change in light intensity incident on a surface with varying voltage instead of resistance as my prerequisite research showed that there was no existing model between the illuminance of an LED and the voltage across it.

2 Hypothesis

It was hypothesised that the correlation between voltage and the luminance was positive. As suggested by the introduction, this hypothesis was forged though no technical knowledge.

3 Background

3.1 Units and Decimal Places (d.p)

All values are in SI units as shown in the Nomenclature, unless otherwise stated. If no unit is present, it is a dimensionless quantity.

Percentages, Gradients, Statistical Data are provided with at least 2 d.p. Raw values are provided at least 3 d.p and 4 d.p for logged values.

3.2 Shockley Diode Equation

The Shockley Diode equation (**TheBellSystemJournal**) is a model for the current across a diode. It is defined as:

$$I_D = I_S \cdot \left(\exp\left(\frac{V_D}{nV_T} \right) - 1 \right) \tag{1}$$

where,

$$V_T = \frac{kT}{q} = 0.02585V \ at \ 300.5K \tag{2}$$

3.3 Ideality factor and luminous efficacy

The ideality factor defines how the profile of the LED deviates from the ideal diode. While the ideality factor can be calculated, it would require understanding and the data points for the quantum properties of the diode, which is beyond the scope of this report[1] (**TheBellSystemJournal**).

[1]The manufacturer was contacted, but they could not provide any of the necessary attributes of the LED

3.4 Luminous efficacy

The measure of how well a light source emits light in the visible spectrum, 380–700 nanometres.

$$\eta = \frac{\phi_{lm}}{P} = \frac{100}{1.1} = 90.\overline{90} \tag{3}$$

Figure 1: Luminous efficacy at 12V (**xavax_pp**)

3.5 Lux and Lumens relationship

Lumens is the amount of luminous flux of a light source, whereas lux measures the illuminance of a surface.

This is important as the data collected from the experiment was in lux and needed to be converted to lumens because of the data point provided by the manufacturer.

$$\phi_{lm} = E_v \cdot A_{sensor} \tag{4}$$

Figure 2: The Relationship between lux and lumens

4 Variables

Independent variables:

Voltage: Manipulated using an AC power supply capable of supplying 0–15V AC. SI Units: Volt (V).

Dependent variables:

Current: This depends on Voltage and the relationship can be found using the Shockley diode equation. SI Units: Amps (A).
Power: Determined by the product of voltage and current. SI Unit: Watt (W).
Luminance: The total luminous output of a light emitting object. SI Unit: Lumens (lm).
Illuminance: The total amount of light incident on a surface. SI Unit: Lux (lx).

Control variables:

Temperature: Changes in Temperature would affect the Thermal Voltage (V_T) and, consequently (I_S), therefore, temperature was controlled by performing all the trials in a short space of time to minimize the effect of temperature variations.
Ambient Luminance: Any environmental luminance could potentially affect the results, which may lead to unaccounted errors. Therefore, a shoebox was used to block out any light that was not from the LED. This was successful as when the lid of the box was closed, 0 illuminance was noted. Thereby validating the control process.

5 Methodology

5.1 Theory

5.1.1 Derivation

By rearranging (3), we get

$$\phi_{lm} = \eta P \tag{5}$$

Furthermore, $P = V_D \cdot I_D$, hence the equation becomes

$$\phi_{lm} = \eta V_D I_D \tag{6}$$

I_D can be broken down further as shown in (1)

$$\phi_{lm} = \eta V_D I_S \left(\exp\left(\frac{V_D}{nV_T} \right) - 1 \right) \tag{7}$$

While I_S can be simplified further, this was omitted for the purposes of this report. See Future Scope

5.1.2 Sample Theoretical Calculations

Due to lack of data given in datasheet, I_S was assumed to be constant and was calculated at $V_D = 12V$ and $I_D = 0.0916A$, which was the only value on the datasheet (**xavax_pp**).

n was also assumed to be equal to η (ideality/emission factor), as they are proportional but no formula exists, and these properties need to be provided by the manufacturer.

Hence, using the Shockley diode equation, I_S can be calculated as:

$$0.0916 = I_S \cdot \left(\exp\left(\frac{12}{2.35} \right) - 1 \right) \tag{8}$$

Rearranging for I_S, yields $I_S = 5.59 \times 10^{-4}$
With this, a sample luminous flux at $V_D = 15$ can be calculated from equation 7

$$\phi_{lm} = 90.\overline{90} \cdot 15 \cdot \left(\exp\left(\frac{15}{2.35} \right) - 1 \right) \tag{9}$$

$$\therefore \phi_{lm} = 449.94 \text{ lm}$$

5.2 Apparatus

- 1x Silicon diode LED
- 1x Laptop with Logger Pro app
- 1x Photometer
- 1x Power supply
- 2x Multimeter
- 8x Alligator clips with wires
- 1x Cardboard box

5.3 Setup

The Setup diagram is shown below, including the circuit on the PCB of the LED (See Appendix), which is shown by the rectifier circuit, capacitor, inductor and led. This was reverse-engineered by looking at the components and the traces on the PCB.

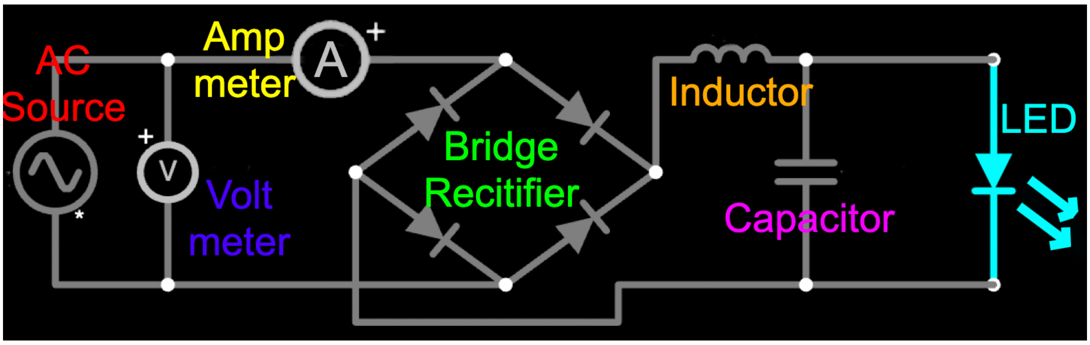

Figure 3: Schematic of the LED (Made by (**IBDPCandidate**), using software from (**CircuitSimulator**))

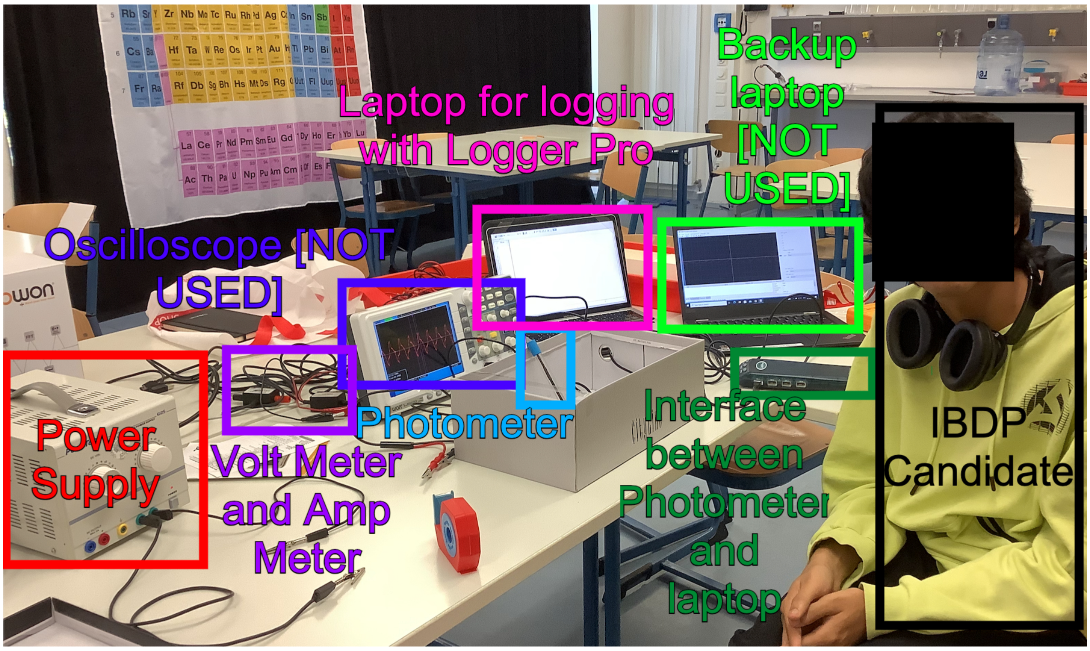

Figure 4: Photo of the setup, on the day of the experiment (**IBDPCandidate**)

5.4 Procedure

1. Set up the circuit as shown in Figure 3
2. Set power supply to the highest value at which the LED does not turn on
3. Start the logging software and turn up the voltage
4. Record the data on the laptop for 10 seconds and then move on to the next voltage until 15V is reached.
5. Repeat step 4, 5 times, with each being a trial, for 5 trials and a total of 600 data points.

6 Safety, Environmental and Ethical

No safety or ethical issues were identified throughout the course of this experiment. Rubber gloves were worn despite operating low AC voltage as sweaty hands could cause mild shock. The setup was taped to the table to prevent any damage to the equipment or any other property. This experiment did not harm the environment to a great extent as all the measuring equipment belonged to the school, thus as no new parts except the LED where bought, the marginal effect to the environment was negligible. The LED, that was purchased for the experiment, was also put to use afterwards to reduce wastage.

7 Data

The table below consolidates the average values of each trial corresponding to the applied voltage. The complete data set can be found in the appendix.

Raw Observed Data			
Voltages (V) $\pm$ 0.1	Average Illumination (lux)	$\pm$ Average Error (lux)	% Error
0	0	0	0
1	0	0	0
2	0	0	0
3	0	0	0
4	193.293	11.013	5.69
5	3898.699	8960.570	229.83
6	41772.866	4500.047	10.77
7	61879.059	9850.934	15.91
8	86162.669	6215.361	7.21
9	94726.656	4489.179	4.73
10	104274.935	3731.749	3.57
11	111992.596	4909.910	4.38
12	119960.788	2603.954	2.17
13	130434.241	4615.974	3.53
14	140951.819	3433.240	2.43
15	149648.108	1504.642	1.00

Table 1: Raw Data Table

The raw data was transformed from lux to lumens to facilitate the use of Equation 7. Given the exponential relationship between voltage and luminance, as established in the background section, the natural logarithm of the luminance was considered for a more comprehensive analysis.

Theoretical and Practical Data					
	Theoretical data		Observed data		
V ± 0.1	Luminance	Ln Luminance	Luminance	Ln Luminance	% Error
0	Omitted	Omitted	0	Undefined	-
1	Omitted	Omitted	0	Undefined	-
2	Omitted	Omitted	0	Undefined	-
3	Omitted	Omitted	0	Undefined	-
4	0.911	-0.0924	0.144	-1.9312	-95.21
5	1.878	0.6306	2.927	1.0741	-41.29
6	3.611	1.2842	31.329	3.4445	-62.71
7	6.637	1.8926	46.409	3.8374	-50.67
8	11.823	2.4700	64.622	4.1685	-40.74
9	20.597	3.0251	71.044	4.2633	-29.04
10	35.293	3.5636	78.206	4.3593	-18.25
11	59.706	4.0894	83.994	4.4307	-7.70
12	100.000	4.6051	89.970	4.4994	2.348
13	166.134	5.1127	97.825	4.5831	11.55
14	274.171	5.6137	105.713	4.6607	20.44
15	449.940	6.1091	112.236	4.7206	29.41
Note: Luminance is measured in lumens				Average Error:	-23.49

Table 2: Processed Data and Error

8 Results

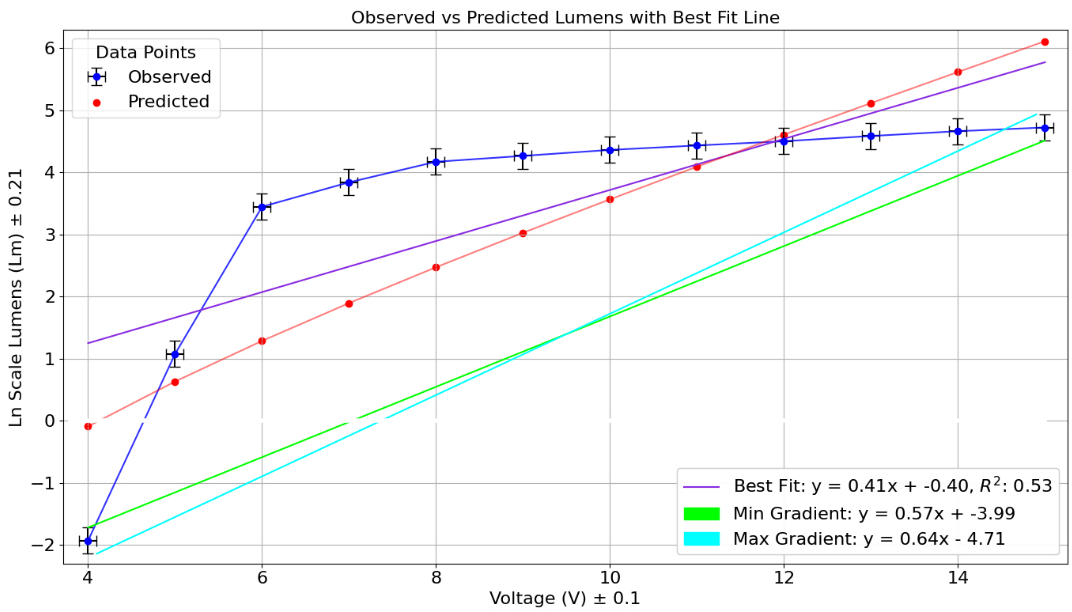

Figure 5: Predicted versus Observed values (**IBDPCandidate**)

The comparison between predicted and observed values[2], as depicted in Figure 5, shows that the expected data underestimates the observed values up to V = 12. This discrepancy can be attributed to various sources of error, which are discussed in the Error section. Beyond V = 12, the expected data overestimates the observed values, likely due to the exponential nature of the relationship.

The data suggests a strong positive correlation between voltage and LED luminance, confirming the hypothesis. However, contrary to the initial visual estimate, the relationship is exponential rather than linear. The line of best fit yielded an equation of y = 0.41x - 0.4 with a goodness of fit of 0.53.

8.1 Statistical Analysis

The Kolmogorov-Smirnov Test was performed on the observed and expected data to validate its statistical significance. The p-value, indicates the probability that the observed data would have occurred by chance if the null hypothesis were true.

In this context, the null hypothesis is that the observed and expected data come from the same distribution. A lower p-value suggests that the observed data significantly deviates from what was expected, leading us to reject the null hypothesis.

Typically, a p-value of less than 0.05 is considered statistically significant (**PValue**). This threshold is widely accepted in the scientific community and indicates that there is less than a 5% probability that the observed results occurred by chance. Therefore, if the p-value obtained from the Kolmogorov-Smirnov Test is less than 0.05, we can conclude that there is a statistically significant difference between the observed and expected data.

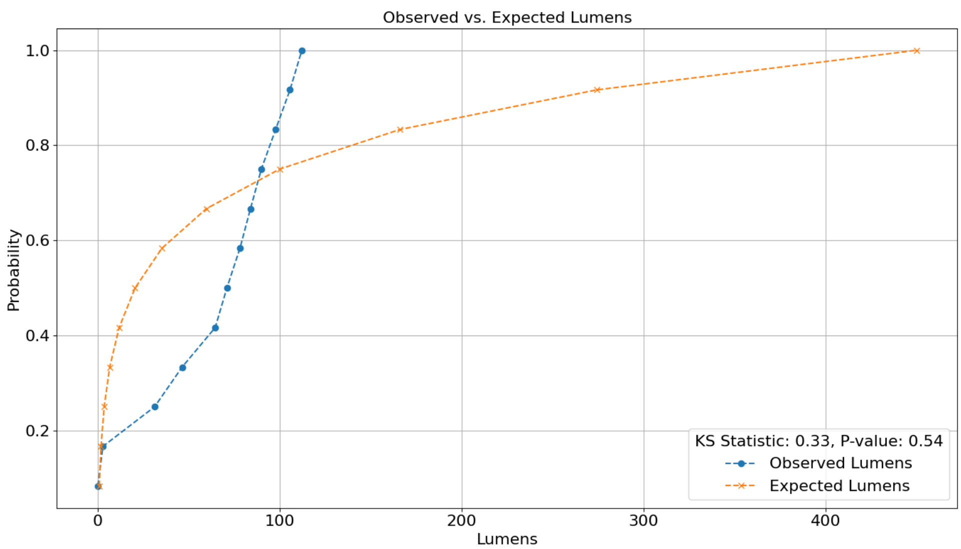

Figure 6: The Kolmogorov-Smirnov Test on the observed and expected lumens (**IBDPCandidate**)

[2]The 0 values have been removed as the Y-axis is a log scale

9 Errors, Assumptions, and Limitations

While it has been established that the data is statistically significant, the possible causes for error and ultimately deviation from the expected values, can be broken down.

9.1 Diode Temperature, Joule Heating and Losses

While the tests were done in a span of 15 minutes with the environmental temperature being relatively constant, due to the resistance of the diode, it was subject to Joule Heating, Therefore, there was loss of power and changing temperature.

The increase in diode temperature, would theoretically lead to a decrease in power draw and therefore, lower luminance, justifying the underestimation of the data.

Similarly, some power could be lost to the wires and resistance of the circuit.

9.2 UV and Infrared Radiation

'Most LEDs emit a narrow band of wavelengths ranging from infrared (at a wavelength of approximately 1000 nanometers) to ultraviolet (about 300 nanometers)' (**LEDInfrared**). As the photometer only measures light in the visible spectrum, energy could have been lost this way, explaining the underestimate.

9.3 Losses due to the bridge rectifier

As shown in Figure 3, the LED, takes input in the form of AC and then converts it to DC. This process is not lossless and led to a lower power output to the LED.

9.4 Quantum Properties of the LED

Due to the fact that I_S is not constant as temperature changes and the ideality factor is not known and was assumed to be equal to $90.\overline{90}$, it could explain the deviation from expected results.

Saturation current can also be theoretically calculated if the quantum properties of the semiconducting material are known. Assuming the diode is an ideal one,

$$I_S = qAn_i^2\left(\frac{1}{N_D}\sqrt{\frac{D_P}{\tau_p}} + \frac{1}{N_A}\sqrt{\frac{D_N}{\tau_N}}\right)$$

9.5 Gaussian Beams and Intensity drop-off

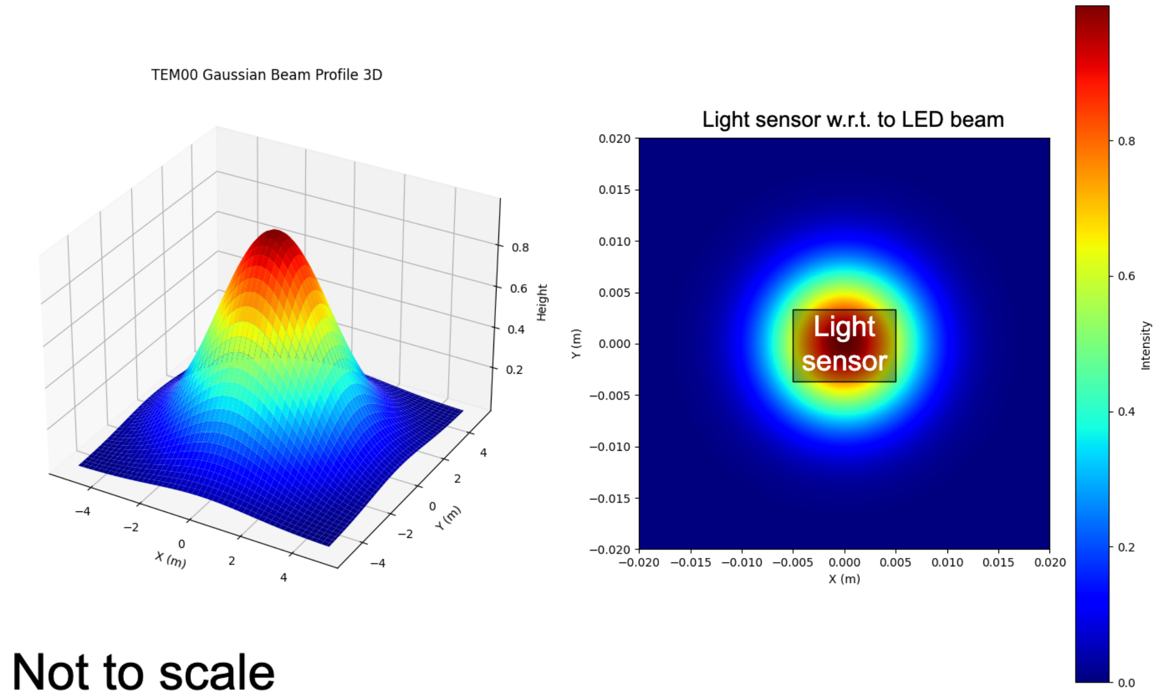

Figure 7: Beam distribution and the photometer (**IBDPCandidate**)

Figure 7 above represents an ideal TEM_{00} monochromatic light source. While a LED was used, Mathematically, a LED is just a continuous combination of different monochromatic sources.

The same figure also demonstrates that, the intensity of the beam and by implication, the illuminance, drops off, the further we go away from the center of beam (X and Y direction) or change the distance between the sensor and the LED (Height).

The second figure on the right shows the light sensor with respect to the beam distribution. Since the sensor is quite small, it could explain the significant underestimate by the theoretical data.

As indicated by Figure 8, the LED follows the TEM_{00}.

Figure 8: Light spread of the LED, displaying the TEM_{00} profile (**IBDPCandidate**)

9.6 Improperly calibrated equipment

Improperly calibrated equipment could lead to systematic errors. This was also a limitation due to cost.

9.7 Floating Point Error

As the values were calculated on a computer, which, at very small values, may lead to inaccurate values, which may have a cascading effect, but it was deemed to be a negligible source of error.

10 Future Scope

This report is not an all encompassing. Due to limitations of cost, infrastructure, complexity, and lack of knowledge, especially of the quantum mechanics of a silicon diode, the theoretical model could not be aligned closer to the real world values.

However, the following things could be explored:

10.1 Gaussian Beams

As shown earlier, the beam of the LED represents the TEM_{00} profile. However, the profile only models monochromatic light as shown below

$$I(r) = I_0 \exp\left(\frac{-2r^2}{\omega^2}\right)$$

where, ω is:

$$\omega = \frac{\lambda}{\pi\theta}$$

However, if the function of the Spectral Power Distribution (See Appendix) is known, and given that:

$$E = \frac{hc}{\lambda}$$

Total intensity can be found using:

$$I_{total} = \int_{\lambda_{min}}^{\lambda_{max}} P(\lambda) d\lambda$$

Where: $P(\lambda)$ is the function of spectral power distribution for a given λ

Saturation current can also be theoretically calculated if the quantum properties of the semiconducting material are known. Assuming the diode is an ideal one,

$$I_S = qAn_i^2 \left(\frac{1}{N_D}\sqrt{\frac{D_P}{\tau_p}} + \frac{1}{N_A}\sqrt{\frac{D_N}{\tau_N}}\right)$$

11 Conclusion

In conclusion, the investigation successfully demonstrated a strong positive correlation between the voltage and luminance of an LED, affirming the initial hypothesis. However, the relationship was found to be exponential rather than linear, disproving the visual estimate.

This report encountered several sources of error that contributed to the deviation from the expected values. These included the increase in diode temperature due to Joule Heating, losses due to the bridge rectifier, and the emission of UV and Infrared radiation that the photometer could not measure. Additionally, the assumption of constant saturation current and the unknown ideality factor could have led to inaccuracies in the results.

The limitations of the study were primarily due to cost constraints and lack of knowledge, particularly in the quantum mechanics of a silicon diode. The equipment used may not have been properly calibrated, leading to potential systematic errors. Furthermore, the small size of the light sensor could have led to a significant underestimate by the theoretical data.

However, this investigation lays the groundwork for future explorations into Gaussian beams. The beam of the LED represents the TEM_{00} profile, but this model only accounts for monochromatic light. Further research could delve into the implications of this profile for LEDs, which are a continuous combination of different monochromatic sources.

While this report has its shortcomings, it demonstrates its validity through statistical analysis, relatively low uncertainty and proper control of background illumination, and appreciation of errors.

A Appendix

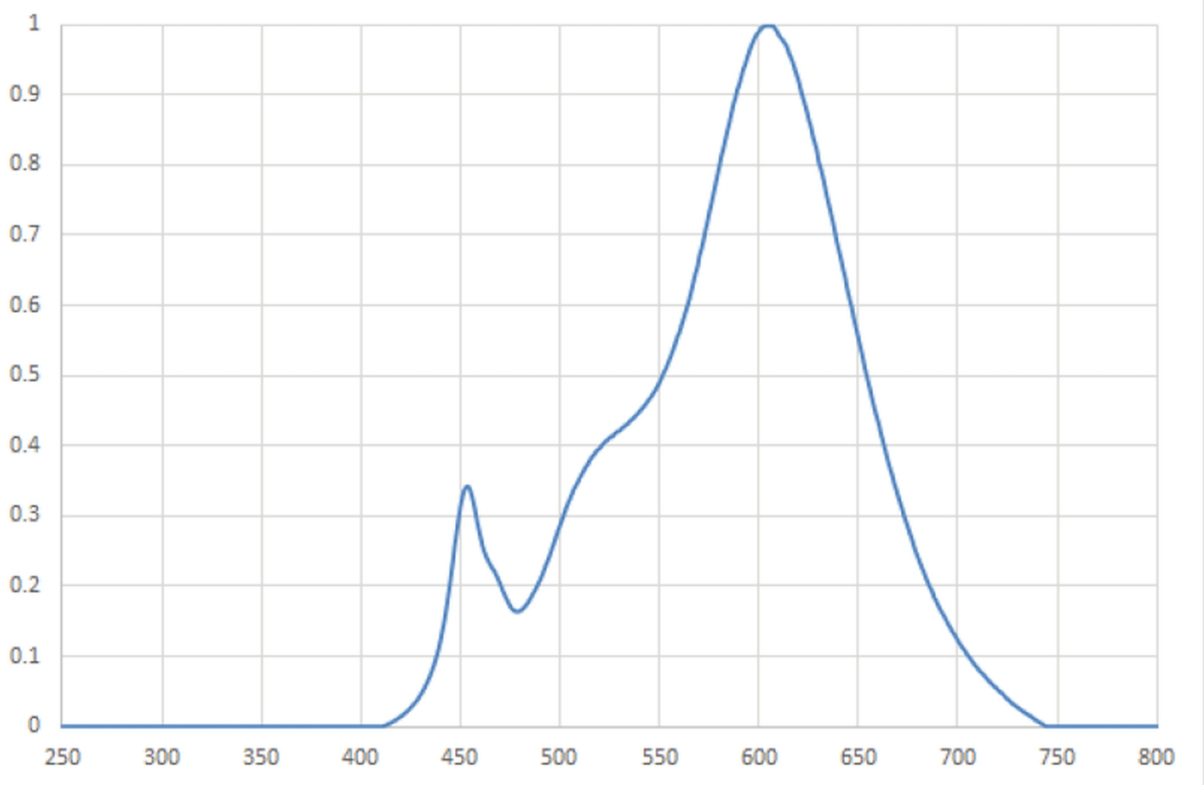

Figure 9: SPD of the LED

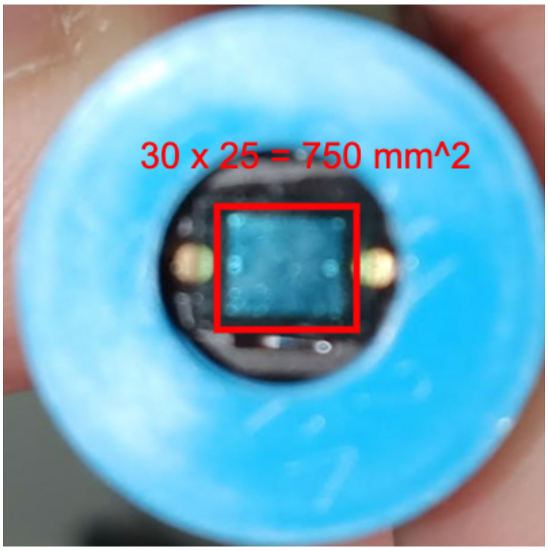

Figure 10: Photometer used

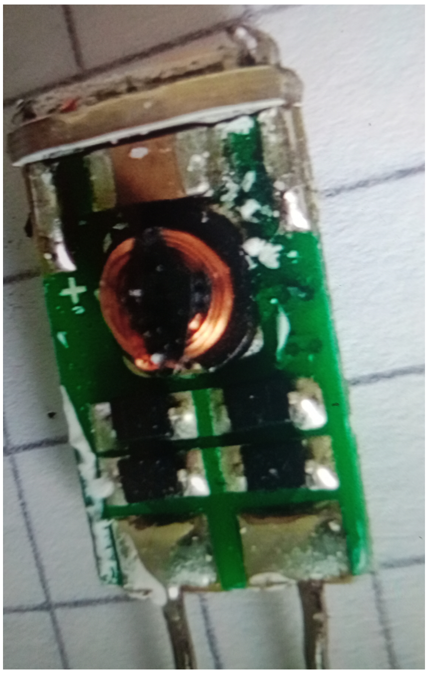

Figure 11: PCB of the LED

Figure 12: Environmental Temperature

		Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial Avg / Sec
V	Time (s)	Illumination (lux)					
0-3	0	0.00	0.00	0.00	0.00	0.00	0.00
4	1	187.66	187.66	187.66	187.66	187.66	187.66
4	2	187.66	187.66	187.66	187.66	187.66	187.66
4	3	187.66	234.58	187.66	187.66	187.66	197.05
4	4	187.66	187.66	187.66	187.66	187.66	187.66
4	5	187.66	187.66	187.66	187.66	187.66	187.66
4	6	187.66	187.66	187.66	187.66	234.58	197.05
4	7	140.75	187.66	234.58	187.66	187.66	187.66
4	8	187.66	140.75	140.75	187.66	234.58	178.28
4	9	234.58	187.66	234.58	234.58	187.66	215.81
4	10	187.66	187.66	187.66	281.49	187.66	206.43
5	11	234.58	234.58	234.58	234.58	140.75	215.81
5	12	187.66	187.66	187.66	187.66	234.58	197.05
5	13	187.66	187.66	234.58	234.58	187.66	206.43
5	14	187.66	187.66	187.66	234.58	187.66	197.05
5	15	187.66	234.58	234.58	234.58	234.58	225.20
5	16	187.66	187.66	234.58	234.58	234.58	215.81

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5	17	187.66	187.66	187.66	234.58	187.66	197.05
5	18	234.58	187.66	234.58	234.58	234.58	225.20
5	19	45,601.99	187.66	187.66	187.66	234.58	9,279.91
5	20	46,212.16	187.66	46,446.55	47,291.11	234.58	28,074.41
6	21	56,017.58	187.66	58,363.37	56,439.48	234.58	34,248.53
6	22	41,098.31	281.49	43,256.20	64,133.88	50,059.17	39,765.81
6	23	65,259.71	281.49	61,225.19	52,029.55	55,360.53	46,831.29
6	24	65,259.71	281.49	49,402.12	48,651.70	50,950.61	42,909.13
6	25	54,610.11	234.58	42,740.17	53,765.55	41,895.61	38,649.20
6	26	40,582.28	234.58	56,955.90	53,483.90	63,570.97	42,965.52
6	27	50,387.70	234.58	51,466.64	48,932.96	62,867.04	42,777.79
6	28	44,898.44	234.58	48,604.82	49,918.54	42,693.29	37,269.94
6	29	31,527.39	234.58	49,824.40	69,763.77	62,163.50	42,702.73
6	30	32,278.04	234.58	84,542.02	83,697.85	47,291.11	49,608.72
7	31	29,885.34	234.58	59,958.35	59,582.94	42,458.90	38,424.02
7	32	32,465.71	65,541.35	64,649.92	84,120.13	78,771.51	65,109.72
7	33	25,662.93	49,214.61	60,380.63	91,391.87	64,462.41	58,222.49
7	34	28,993.94	43,068.69	77,551.93	67,464.86	68,215.66	57,059.02
7	35	33,357.10	53,765.55	69,763.77	59,723.58	82,571.64	59,836.33
7	36	63,476.83	50,809.59	78,818.38	81,774.34	65,775.74	68,130.98
7	37	63,007.68	51,701.03	63,899.11	93,643.90	79,100.03	70,270.35
7	38	92,658.71	43,725.36	83,040.80	69,294.61	64,227.64	70,589.42
7	39	21,112.11	53,296.39	63,945.99	90,594.19	71,687.27	60,127.19
7	40	87,967.14	47,619.63	73,517.02	76,941.75	69,059.84	71,021.08
8	41	76,425.72	52,217.06	92,330.18	77,598.80	78,677.75	75,449.90
8	42	86,137.39	93,268.50	98,663.99	96,552.60	98,194.84	94,563.46
8	43	79,850.45	77,035.51	77,082.39	103,824.72	77,551.93	83,069.00
8	44	101,009.78	86,371.77	103,120.79	76,941.75	88,483.17	91,185.45
8	45	88,342.54	60,098.98	82,102.48	80,836.03	77,410.91	77,758.19
8	46	80,413.75	85,058.44	90,406.68	94,535.34	76,519.47	85,386.73
8	47	76,848.00	84,589.28	93,737.65	81,868.10	91,485.62	85,705.73
8	48	94,910.74	83,979.11	97,397.16	88,295.28	98,992.14	92,714.88

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8	49	83,369.32	68,872.33	76,144.46	109,313.59	92,330.18	86,005.98
8	50	100,352.73	80,225.85	105,372.82	85,339.70	77,645.68	89,787.36
9	51	86,981.95	75,956.56	82,055.60	104,340.76	92,001.66	88,267.31
9	52	90,406.68	74,361.58	107,249.45	84,964.30	85,292.83	88,454.97
9	53	103,918.48	92,986.85	89,937.52	105,466.58	82,243.50	94,910.58
9	54	102,369.99	78,396.10	106,780.30	84,260.76	96,224.07	93,606.24
9	55	89,608.99	77,270.28	100,258.97	101,150.41	102,933.29	94,244.39
9	56	102,980.16	92,330.18	95,895.93	104,903.67	107,624.86	100,746.96
9	57	94,488.46	91,579.38	103,730.58	106,029.49	90,641.06	97,293.79
9	58	90,406.68	98,053.82	83,744.72	93,127.86	105,138.05	94,094.23
9	59	111,424.99	82,618.52	100,165.22	88,483.17	85,105.32	93,559.44
9	60	101,478.94	95,754.91	111,331.23	105,654.09	96,224.07	102,088.65
10	61	119,541.29	80,085.22	91,063.34	102,557.88	98,006.94	98,250.94
10	62	106,874.05	101,948.09	110,064.40	114,615.33	110,298.78	108,760.13
10	63	107,624.86	93,737.65	98,663.99	96,881.12	111,096.46	101,600.82
10	64	107,436.96	101,525.81	101,384.80	111,612.50	107,343.21	105,860.66
10	65	110,627.31	106,920.93	109,548.36	91,157.48	111,143.34	105,879.48
10	66	103,214.55	92,799.34	118,696.73	113,066.85	92,658.71	104,087.23
10	67	91,344.99	103,308.30	101,900.83	111,190.22	99,836.69	101,516.21
10	68	110,158.15	83,791.60	110,439.80	95,145.12	102,557.88	100,418.51
10	69	107,014.68	97,537.79	103,261.43	119,635.05	106,827.17	106,855.22
10	70	122,403.11	110,862.08	115,084.49	96,505.72	102,745.39	109,520.16
11	71	114,709.09	93,972.42	109,548.36	112,316.43	102,651.64	106,639.59
11	72	114,380.56	118,509.22	122,731.64	116,820.10	119,681.92	118,424.69
11	73	121,746.44	106,686.54	109,782.75	110,533.55	96,036.56	108,957.17
11	74	116,866.98	91,298.11	111,049.59	114,474.32	121,887.08	111,115.21
11	75	120,245.22	115,788.03	120,151.08	94,253.69	126,109.49	115,309.50
11	76	97,209.26	107,436.96	120,526.48	110,345.66	100,165.22	107,136.72
11	77	110,392.92	102,557.88	93,456.01	110,533.55	116,022.81	106,592.63
11	78	114,568.46	112,926.21	109,172.96	114,474.32	107,343.21	111,697.03
11	79	132,443.30	112,410.18	110,580.43	104,809.91	108,140.89	113,676.94
11	80	127,000.93	93,503.27	126,672.40	133,381.62	121,324.16	120,376.47

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12	81	130,331.90	124,092.23	105,748.23	105,138.05	127,610.71	118,584.23
12	82	131,035.83	117,101.75	116,304.07	132,349.16	121,605.81	123,679.32
12	83	122,168.72	111,612.50	134,225.79	122,872.27	115,553.65	121,286.59
12	84	105,701.35	116,679.47	124,420.75	135,070.35	123,482.44	121,070.87
12	85	127,141.56	98,147.58	119,025.26	123,529.32	118,978.38	117,364.42
12	86	112,081.66	121,933.95	118,368.59	131,974.14	126,766.54	122,224.98
12	87	124,983.67	122,168.72	116,445.08	104,763.04	116,726.35	117,017.37
12	88	115,788.03	97,162.38	125,968.86	117,430.28	129,299.83	117,129.88
12	89	130,285.03	115,084.49	103,918.48	136,196.56	130,331.90	123,163.29
12	90	116,350.95	117,570.91	134,132.04	117,946.31	104,434.51	118,086.94
13	91	140,793.99	110,955.83	121,746.44	131,082.71	117,054.87	124,326.77
13	92	117,570.91	107,765.49	139,480.66	128,361.52	138,307.57	126,297.23
13	93	139,292.76	131,176.46	127,704.85	138,823.61	137,838.42	134,967.22
13	94	131,317.09	124,795.77	128,502.15	138,823.61	132,677.69	131,223.26
13	95	118,086.94	114,568.46	139,902.94	139,433.78	139,949.81	130,388.39
13	96	135,914.91	119,822.94	119,822.94	127,235.70	135,445.75	127,648.45
13	97	137,744.66	105,982.61	123,623.07	128,924.43	127,047.80	124,664.52
13	98	138,964.62	126,813.42	137,369.26	130,801.06	125,875.10	131,964.69
13	99	140,090.45	134,085.16	126,156.37	138,729.85	139,808.80	135,774.12
13	100	141,216.27	133,240.60	136,243.44	148,629.03	126,109.49	137,087.76
14	101	146,330.12	123,810.58	142,342.48	140,465.85	130,613.55	136,712.52
14	102	137,838.42	139,574.41	146,705.52	140,231.08	137,697.78	140,409.44
14	103	147,034.05	139,574.41	145,626.58	140,653.36	146,189.49	143,815.58
14	104	144,219.10	129,206.08	131,927.27	139,433.78	145,720.33	138,101.31
14	105	142,576.86	136,102.42	143,468.30	141,920.20	133,287.48	139,471.05
14	106	139,058.38	116,726.35	145,016.40	148,112.99	144,641.38	138,711.10
14	107	146,142.61	120,761.25	139,902.94	142,436.23	140,043.57	137,857.32
14	108	144,594.51	129,628.36	148,112.99	148,253.62	140,793.99	142,276.69
14	109	151,256.46	139,105.25	143,139.77	151,397.09	147,503.20	146,480.36
14	110	151,256.46	125,780.96	151,209.58	151,256.46	148,910.68	145,682.83
15	111	151,397.09	131,598.74	151,537.72	151,490.85	142,342.48	145,673.38

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15	112	151,162.71	147,550.08	151,397.09	151,443.97	151,350.21	150,580.81
15	113	151,256.46	145,344.93	151,397.09	151,162.71	151,256.46	150,083.53
15	114	151,209.58	141,357.29	151,537.72	151,537.72	151,537.72	149,436.01
15	115	151,443.97	146,799.28	151,397.09	151,350.21	151,303.34	150,458.78
15	116	151,209.58	145,251.17	151,397.09	151,350.21	151,443.97	150,130.41
15	117	151,350.21	142,201.46	151,631.86	151,256.46	151,397.09	149,567.42
15	118	151,256.46	139,949.81	151,537.72	151,631.86	151,209.58	149,117.09
15	119	151,162.71	147,643.84	151,584.99	151,303.34	151,537.72	150,646.52
15	120	151,350.21	148,066.12	151,443.97	151,537.72	151,537.72	150,787.15

```
# This code has been formatted using Black for easier readability and
    comprehension
# https://github.com/psf/black

import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib import patches

# Taken from:
# https://stackoverflow.com/questions/52653734/how-to-do-a-3d-plot-of-
    gaussian-using-numpy
# Gaussian parameters for the first plot
A = 1
x0 = 0
y0 = 0
sigma_X = 2
sigma_Y = 2

xg = np.linspace(-5, 5, num=100)
yg = np.linspace(-5, 5, num=100)
X, Y = np.meshgrid(xg, yg)

theta = np.pi
a = np.cos(theta)**2 / (2 * sigma_X**2) + np.sin(theta)**2 / (2 *
    sigma_Y**2)
b = -np.sin(2 * theta) / (4 * sigma_X**2) + np.sin(2 * theta) / (4 *
    sigma_Y**2)
c = np.sin(theta)**2 / (2 * sigma_X**2) + np.cos(theta)**2 / (2 *
    sigma_Y**2)

# Calculate the Gaussian distribution
aXXdet = a * (X - x0)**2
bbXYdet = 2 * b * (X - x0) * (Y - y0)
cYYdet = c * (Y - y0)**2
Z = A * np.exp(-(aXXdet + bbXYdet + cYYdet))

# Parameters for the second plot
wavelength = 0.5e-6  # Wavelength (in meters)
beam_waist = 0.01  # Beam waist (in meters)
grid_size = 1000  # Grid size for plotting

# Define grid for the second plot
x = np.linspace(-5 * beam_waist, 5 * beam_waist, grid_size)
y = np.linspace(-5 * beam_waist, 5 * beam_waist, grid_size)
X2, Y2 = np.meshgrid(x, y)

# Calculate Gaussian beam intensity profile
w0 = beam_waist / np.sqrt(2)
R = np.sqrt(X2**2 + Y2**2)
Gaussian_beam = np.exp(-(R**2) / w0**2)

# Plotting both figures in subplots
fig = plt.figure(figsize=(10, 6))


ax1 = fig.add_subplot(121, projection='3d')
ax1.plot_surface(X, Y, Z, cmap=cm.jet)
```

```
ax1.set_xlabel('X (m)')
ax1.set_ylabel('Y (m)')
ax1.set_zlabel('Height')
ax1.set_title('TEM00 Gaussian Beam Profile 3D')
# ---------END----------

# This is made by me
# plot 2
ax2 = fig.add_subplot(122)
img = ax2.imshow(Gaussian_beam, cmap=cm.jet, extent=[-5 * beam_waist, 5
    * beam_waist, -5 * beam_waist, 5 * beam_waist])
fig.colorbar(img, ax=ax2, label='Intensity')
ax2.set_xlabel('X (m)')
ax2.set_ylabel('Y (m)')
ax2.set_title('TEM00 Gaussian Beam Profile 2D')

# Add Rectangle in 2D plot
rect2 = patches.Rectangle((-0.005,-(beam_waist/np.e)), beam_waist, 0.7*
    beam_waist,
                           linewidth=1, edgecolor='black', facecolor=(0,
    0, 0, 0.3))
ax2.add_patch(rect2)

# Set limits to zoom in and center around a specific area
zoom_factor = 2  # Change this factor to adjust the zoom level
center_x, center_y = 0, 0  # Center coordinates for zoom

x_start = center_x - beam_waist * zoom_factor
x_end = center_x + beam_waist * zoom_factor
y_start = center_y - beam_waist * zoom_factor
y_end = center_y + beam_waist * zoom_factor

ax2.set_xlim(x_start, x_end)
ax2.set_ylim(y_start, y_end)

plt.tight_layout()
plt.show()
```

Listing 1: Gaussian Distribution Diagram

```python
# This code has been formatted using Black for easier readability and comprehension
# https://github.com/psf/black

import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import ks_2samp

plt.rc('font', size=16)
plt.rc('axes', titlesize=16)
plt.rc('axes', labelsize=16)
plt.rc('xtick', labelsize=16)
plt.rc('ytick', labelsize=16)
plt.rc('legend', fontsize=16)
plt.rc('figure', titlesize=16)


observed_lumens = [0.144969796, 2.927543071, 31.32964957, 46.40929451, 64.62200192, 71.04499234, 78.20620195, 83.9944471, 89.97059138, 97.82568078, 105.7138648, 112.2360813]
expected_lumens = [0.911709, 1.87877, 3.61185, 6.63714, 11.8235, 20.5979, 35.2932, 59.7069, 100, 166.134, 274.171, 449.94]

sorted_observed = np.sort(observed_lumens)
sorted_expected = np.sort(expected_lumens)

ecdf_observed = np.arange(1, len(sorted_observed) + 1) / len(sorted_observed)
ecdf_expected = np.arange(1, len(sorted_expected) + 1) / len(sorted_expected)

statistic, p_value = ks_2samp(observed_lumens, expected_lumens)

plt.figure(figsize=(10, 6))
plt.plot(sorted_observed, ecdf_observed, marker='o', linestyle='dashed', label='Observed Lumens')
plt.plot(sorted_expected, ecdf_expected, marker='x', linestyle='dashed', label='Expected Lumens')
plt.xlabel('Lumens')
plt.ylabel('Probability')
plt.title('Observed vs. Expected Lumens')
plt.legend(title=f'KS Statistic: {statistic:.2f}, P-value: {p_value:.2f}')
plt.grid(True)
plt.tight_layout()
plt.show()
```

Listing 2: Kolmogorov-Smirnov Test Code

```python
# This code has been formatted using Black for easier readability and comprehension
# https://github.com/psf/black

import matplotlib.pyplot as plt
import numpy as np
from matplotlib.patches import Patch
from sklearn.metrics import r2_score

plt.rc('font', size=16)
plt.rc('axes', titlesize=16)
plt.rc('axes', labelsize=16)
plt.rc('xtick', labelsize=16)
plt.rc('ytick', labelsize=16)
plt.rc('legend', fontsize=16)
plt.rc('figure', titlesize=16)

voltages = np.array([4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15])
observed_lumens = np.array([-1.93122986, 1.074163529, 3.44456492, 3.837499752, 4.16855494,
                            4.263313371, 4.359348953, 4.430750691, 4.499482854, 4.583187127,
                            4.660736055, 4.720604522])
predicted_lumens = np.array([-0.09243441877, 0.6306173074, 1.284220106, 1.892681148, 2.470089076,
                             3.025189129, 3.563690311, 4.089447592, 4.605170186, 5.112794692,
                             5.613751999, 6.109114241])

y_errors = np.full_like(observed_lumens, 0.21)
x_errors = np.full_like(observed_lumens, 0.1)

new_point1 = (4, -1.72122986)
new_point2 = (15, 4.510604522)

min_grad = (new_point2[1] - new_point1[1]) / (new_point2[0] - new_point1[0])
min_grad_intercept = new_point1[1] - min_grad * new_point1[0]

max_grad = (np.max(observed_lumens) + np.max(y_errors)) - (np.min(observed_lumens) - np.min(y_errors))

max_grad_volt = np.array([voltages[np.argmax(observed_lumens)] - 0.1, voltages[np.argmin(observed_lumens)] + 0.1])

max_grad_intercept = observed_lumens[np.argmax(observed_lumens)] - max_grad * max_grad_volt[0]

index_y_equals_zero = np.where(observed_lumens == 0)[0]
observed_lumens[index_y_equals_zero] = np.nan
y_errors[index_y_equals_zero] = np.nan

plt.figure(figsize=(10, 6))

observed = plt.errorbar(
    voltages,
    observed_lumens,
    xerr=x_errors,
```

```
    yerr=y_errors,
    fmt='o',
    label='Observed',
    color='blue',
    ecolor='black',
    capsize=5,
    marker='o',
    mec='blue',
)
predicted = plt.scatter(voltages, predicted_lumens, label='Predicted', color='red')

plt.plot(voltages, observed_lumens, linestyle='-', color='blue', alpha=0.8, label='_nolegend_')
plt.plot(voltages, predicted_lumens, linestyle='-', color='red', alpha=0.5, zorder=4)

m, b = np.polyfit(voltages, observed_lumens, 1)
r_squared = r2_score(observed_lumens, predicted_lumens)
line = plt.plot(voltages, m * voltages + b, label=f'Best Fit: y = {m:.2f}x + {b:.2f}, $R^2$: {r_squared:.2f}', color='blueviolet')


plt.plot([new_point1[0], new_point2[0]], [new_point1[1], new_point2[1]], label=f'Min Gradient: y = {min_grad:.2f}x + {min_grad_intercept:.2f}', color='lime', linestyle='-')

plt.plot(max_grad_volt, [np.max(observed_lumens) + np.max(y_errors), np.min(observed_lumens) - np.min(y_errors)], label=f'Max Gradient: y = 0.64x - 4.71', color='cyan', linestyle='-')

plt.xlabel('Voltage (V) $ \pm $ 0.1') # changed to \pm for latex compatibility
plt.ylabel('Ln Scale Lumens (Lm) $ \pm $ 0.21')
plt.title('Observed vs Predicted Lumens with Best Fit Line')
plt.grid(True)

legend1 = plt.legend(handles=[observed, predicted], loc='upper left', title='Data Points')


legend2 = plt.legend(
    handles=[
        line[0],
        Patch(color='lime', label=f'Min Gradient: y = {min_grad:.2f}x + {min_grad_intercept:.2f}'),
        Patch(color='cyan', label=f'Max Gradient: y = 0.64x - 4.71'),
    ],
    loc='lower right',
)

#Ln 0 is indefined
plt.plot([15,3.8], [0,0], color='white', linestyle='-', linewidth=4, zorder=10)

plt.gca().add_artist(legend1)
plt.gca().add_artist(legend2)

```

```
plt.tight_layout()
plt.show()
```

Listing 3: Data Plot